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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Derivation of the Apollo Unified
S-Band Communication System Pseudo-
Random Range Code Spectrum

TM- 67-2034-8**DATE-** December 28, 1967**FILING CASE NO(S)-** 320**AUTHOR(S)-** W. J. Benden

FILING SUBJECT(S)- Derivation of Apollo
(ASSIGNED BY AUTHOR(S)- Range Code Spectrum

ABSTRACT

The Apollo Unified S-Band System uses a Pseudo-Random Code for Ranging (sometimes designated Pseudo-Random Noise). The spectrum of this code is of interest in the overall link and in its effects on the Lunar Module (LM) spacecraft tracking antenna.

The fundamental correlation components, which add to form the composite Apollo Pseudo-Random Range Code correlation function, are determined. The spectrum is found from these components.

Starting with the basic properties of Pseudo-Noise (PN) Sequences, some experimental data on the composite correlation function, and through the use of Karnaugh Charts, the correlation components are determined. The spectrum is then found through the Wiener-Khintchine relationship.

Calculated results are compared to measured spectral data. The experimental data is somewhat limited in that only the higher level components are clearly evident. The very close agreement in these cases provides confidence that the calculated results are also correct for the lower level components.

(NASA-CR-93403) DERIVATION OF THE APOLLO
UNIFIED S-BAND COMMUNICATION SYSTEM
PSEUDO-RANDOM RANGE CODE SPECTRUM (Bellcomm,
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BELLCOMM, INC.

SUBJECT: Derivation of the Apollo Unified
S-Band Communication System Pseudo
Random Range Code Spectrum
Case 320

DATE: December 28, 1967
FROM: W. J. Benden
TM - 67-2034-8

TECHNICAL MEMORANDUM

I. INTRODUCTION

Excessive jitter has been noted in the Lunar Module R-F Tracking Antenna during compatibility testing at the Manned Spacecraft Center, Houston, Texas. It has been shown¹ that a major portion of this phenomenon is brought about by the mixing of Apollo Range Code frequency components with Up-Voice and Up-Data subcarrier frequencies.* The range code frequencies which prove to be the most detrimental are multiples of 2Hz and 0.18 Hz¹. Of course, fundamental to the analysis of such a problem, is the entire spectrum generated by this pseudo-random code.

With the exception of Reference 1 and Reference 2, there appears to be little published concerning the Apollo Range Code spectrum itself. Reference 1 is concerned with the spectral components appearing in the spacecraft's receiver and its Coherent Amplitude Detector output and does not go into an actual derivation of the range code spectral components. Appendix C of Reference 2 contains a very brief description on the spectral components.

It is the purpose of this memorandum to derive the Apollo Pseudo-Random Range Code Baseband Spectrum as it appears at the input to the MSFN transmitter. This is accomplished from the information contained in Reference 3 and from the experimental data contained in Reference 4. Although, the experimental work includes somewhat limited measurements on correlation, as well as spectral components, it contains enough information to spot check the results obtained in this memorandum.

The spectrum derived is useful in the analysis of the LM R-F Tracker problem, and may be useful in other communication analyses on the Apollo Program and future manned space flight programs.

*Incidental amplitude modulation is also involved, but is not of the same mechanism as the code effects and is not considered herein.

II. THE APOLLO PSEUDO-RANDOM CODE

The JPL Mark 1* ranging subsystem, used in the Apollo Unified S-Band System, determines range from the time delay of a digital sequence; the delay being introduced by transmitting an R-F carrier, modulated by the sequence, to and from the spacecraft. This binary sequence is clocked at approximately 1 MHz and belongs to a family of codes called Pseudo-Random, or Pseudo-Noise (PN), sequences.

The Apollo Code is made up of the following components:

<u>Component</u>	<u>Bit Length</u>	<u>Type of Code</u>
C ₁	2	Clock
X	11	Barker Code
A	31	Maximal Length
B	63	Shift Register
C	127	Codes

When the above components are combined in accordance with the Boolean expression

$$C_1 \oplus \bar{X} [AB + AC + BC]**$$

an overall code period of approximately five and one-half million bits results. (i.e., the product of the sequence lengths.) Thus, the code period is greater than the earth-moon round trip propagation time.

III. PN SEQUENCE CHARACTERISTICS

PN sequences exhibit the following basic characteristics:³

1. As shown in Figure 1 PN sequences (binary) have two-level autocorrelation functions.

*See Reference 5 and the bibliography in Reference 6 for additional JPL documents.

**Sometimes shown in an equivalent form of

$$X C_1 + \bar{X} [(AB + BC + AC) \oplus C_1]$$

2. In length "L", there are $\frac{(L-1)}{2}$ "zeros" and $\frac{(L+1)}{2}$ "ones" or vice versa.
3. Sequences formed by logical operations on two or more component PN sequences can exhibit multiple peaks in the autocorrelation function. However, the composite waveform can be broken into simpler waveforms each of which can be transformed to the frequency domain and superimposed (see page 77, Reference 3).
4. The normalized autocorrelation function (ρ) can be determined from a Karnaugh Chart* by simply using an agreement-disagreement relation, i.e.

$$\rho = \frac{\text{number agreements} - \text{number disagreements}}{\text{number agreements} + \text{number disagreements}}$$

5. By combining a PN sequence with a clock signal, having a period equal to two PN bit intervals, the correlation function exhibits inverted (negative) portions when the PN components are in phase and the clock is one-half period out of phase, (relative to $\tau = 0$).

IV. AUTOCORRELATION FUNCTION

A measured autocorrelation function for the Apollo Lunar Code is shown in Figure 2.⁴ Although the number of bit intervals shown is relatively small (compared to the length of the combined Code), definite characteristics are seen. Pronounced peaks occur at 11 bit intervals indicating an "X" component contribution and are alternately positive and negative as mentioned in item 5 above. Notice peaks occurring at 31 bit intervals - contribution due to the A component of the code and again alternately positive and negative. With this information, (and that shown on Figure 1) one can sketch, as in Figure 3, the correlation function for both X and A without any long rigorous mathematical equations.**

*The negative portion shown in Figure 1 has been neglected.

**In this memorandum the probabilities are assumed equal in the chart.

This measured correlation function has been determined by comparing the locally generated code with that received from the spacecraft. The spectral data to be used in later paragraphs, however, is MSFN baseband data, i.e., before modulation. Thus, it is not subject to any modulation or de-modulation parameters normally involved in transmitting to and from the spacecraft.

Since there are five basic code components (see page 2) forming the Apollo Range Code there exists the possibility of having the following individual waveforms adding together to form the composite correlation waveform:

C ₁	C ₁ AB
C ₁ X	C ₁ AC
C ₁ A	C ₁ BC
C ₁ B	C ₁ XAB
C ₁ C	C ₁ XAC
C ₁ XA	C ₁ XBC
C ₁ XB	C ₁ ABC
C ₁ XC	C ₁ XABC

The Karnaugh Chart can be used to determine the peak value (which can be zero if the fundamental component is nonexistent) for each of the above listed possible correlation components. Of course, the individual correlation waveforms must add to "one" at $\tau = 0$.

Karnaugh Charts - Figure 4 illustrates the calculation of the "clock" correlation component. Other components are calculated in the same manner; keeping in mind that their sum must add to one (1) at $\tau = 0$. Cross-correlation between these components is negligible.⁷ Table 1 summarizes the results obtained from the Karnaugh charts.

IV. PSEUDO-RANDOM RANGE CODE SPECTRUM

Having determined the correlation components (Table 1) the corresponding spectrum is found easily from the Fourier Transform*

$$X(f) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(\tau) e^{-j\omega_0 \tau} d\tau$$

Since the spectrum is found in such a straightforward manner, only the "X" spectral components will be derived and the others simply listed in Table 2. However, the results obtained will be compared with a spectrum analyzer plot (Reference 4) to confirm their validity. Some of the other components derived cannot be validated from the analyzer plot because they are of such a low level they cannot be distinguished.

These low-level very low frequency components prove to be the most detrimental to the LM-RF track system as mentioned in the Introduction.

Transform of the "X" component. Referring to Figure 3 it is seen that the "X" correlation waveform can be broken into two components: a positive waveform called $y(\tau)$ and a negative waveform called $z(\tau)$. Consider just $y(\tau)$ as shown on Figure 5a and its derivatives shown in Figure 5b and c. The waveform shown in c is now in a most convenient form for transforming to the frequency domain. Thus proceeding with the transform of these delta functions, and integrating twice, we have:

$$F\{\ddot{Y}(\tau)\} = \frac{2k}{Tp} \left[\frac{e^{j\omega_0 p}}{2} + \frac{e^{-j\omega_0 p}}{2} - 1 \right] \quad (1)$$

where $\omega_0 = 2\pi f_0$ (fundamental frequency of $Y(\tau)$)

F = Fourier Transform

p = One bit interval

*In the designation "x component" the clock operation on original "x" sequence is implied.

$$F\{Y(\tau)\} = \frac{1}{(jnw_o)^2} F\{\ddot{Y}(\tau)\}$$

and

$$\frac{1}{(jnw_o)^2} \cdot F\{\ddot{Y}(\tau)\} = \frac{2k(1 - \cos nw_o p)}{T p n^2 w_o^2} \quad (2)$$

$$= \frac{4k(\sin^2 nw_o p/2)}{2\ell_x p^2 n^2 w_o^2} = \frac{k \sin^2 A}{2\ell_x A^2} \quad (2)$$

where $A = \frac{n\pi}{2\ell_x}$, $T = 2\ell_x p$

Except for a sign change and a shift of llp , $Z(\tau)$ transforms in the same manner. Thus

$$F\{Z(\tau)\} = \frac{-k}{2\ell_x} \frac{\sin^2 A}{A^2} e^{-jnw_o llp} \quad (3)$$

Combining equations (2) and (3)

$$F\{X(\tau)\} = \frac{k}{2\ell_x} \frac{\sin^2 A}{A^2} (1 - e^{-jn\pi}) \quad (4)$$

Examination of (4) shows $(1 - e^{-jn\pi})$ to be zero when n is even, i.e., $n=0, \pm 2, \pm 4, \pm 6, \dots$. When not zero, it is 2. Therefore equation (4) can be written as:

$$F\{X(\tau)\} = \frac{k}{\ell_x} \frac{\sin^2 \frac{n\pi}{2\ell_x}}{\left(\frac{n\pi}{2\ell_x}\right)^2}, \quad n = \pm 1, \pm 3, \pm 5, \dots \quad (5)$$

$$= \frac{1}{4\ell_x} \frac{\sin^2 \frac{n\pi}{2\ell_x}}{\left(\frac{n\pi}{2\ell_x}\right)^2}, \quad \text{odd } n \quad (6)$$

since $k = \frac{1}{4}$, from Table 1.

Equation (6) is sketched on Figure 6. The measured data on Figure 7 agrees with Figure 6, in that the spectrum follows a $\frac{\sin X}{X}$ type envelope and the spectral lines appear at odd multiples of 45 KHz which is the fundamental frequency of the X component. Also evident in Figure 7 is the clock component appearing at about 496 KHz. If a clock component is considered in Equation (6) there should be spectral lines appearing at odd multiples of 496 KHz. Figure 8 shows a pronounced spectral line at approximately 3×496 KHz (about 1500 KHz).

It follows that equation (6) can be generalized to account for all spectral components arriving from Table 1. Thus,

$$F\{Q(\tau)\} = \frac{k}{\ell_{\phi}} \frac{\sin^2 \frac{n\pi}{2\ell_{\phi}}}{\left(\frac{n\pi}{2\ell_{\phi}}\right)^2}, \quad n \text{ odd} \quad (7)$$

where

$Q(\tau)$ - represents the components indicated on Table I.

ℓ_{ϕ} - represents the sequence lengths given on page 2.

Figures 9, 10, and 11 illustrate the spectral make up of these components and indicates their relative amplitudes. For convenience, the envelope amplitude for the clock signal at $f=0$ is taken as a reference and the other envelopes expressed in dB down from the clock envelope.

SUMMARY

Table 2, in essence, summarizes the results of this memorandum. It lists the fundamental frequency components appearing in the Apollo Pseudo-Random-Range Code spectrum as well as their relative amplitudes. The code is such that it produces a spectrum which can be considered as due to the combination of the various code elements. These spectra

have a $\frac{\sin^2 X}{X^2}$ envelope and have frequency components related to the length of the code combinations. The longer the code, the lower (more closely spaced) the frequencies and the corresponding lower amplitudes.

It is these low frequencies which have been of concern in the studies of the LM-RF tracker problem.

William Benden
W. J. Benden

2034-WJB-ew

Attachments
Tables I & II
Figures 1 - 11

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TABLE I

CORRELATION COMPONENTS* WHICH ADD TO FORM
COMPOSITE CORRELATION FUNCTION OF THE
APOLLO PSEUDO-RANDOM RANGE CODE

<u>Component</u>	<u>Peak Amplitude</u>
C l	1/4
C lX	1/4
C lA	1/16
C lB	1/16
C lC	1/16
C lXA	1/16
C lXB	1/16
C lXC	1/16
C lABC	1/16
C lXABC	1/16

*Cross correlation between components has been assumed negligible. If considered, there would be very small components resulting from C l AB, C l AC, C l BC, C l XAB, C l XAC, and C l XBC. (see Reference 7)

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TABLE 2

SPECTRAL COMPONENTS OF THE APOLLO
PSEUDO-RANDOM RANGE CODE
AT $f=0$

<u>Code Component</u>	<u>Amplitude Relative to Clock Envelope</u>	<u>Spectral Lines Occur at $(2n+1) f_o$, f_o Equals</u>
C ℓ	- 0.0 dB	496.4 KHZ
C ℓ X	- 10.4 dB	45.1 KHZ
C ℓ A	- 21.0 dB	16.0 KHZ
C ℓ B	- 24.0 dB	7.87 KHZ
C ℓ C	- 27.0 dB	3.90 KHZ
C ℓ XA	- 31.3 dB	1.45 KHZ
C ℓ XB	- 34.5 dB	716.0 HZ
C ℓ XC	- 37.5 dB	355.0 HZ
C ℓ ABC	- 60.0 dB	2.0 HZ
C ℓ XABC	- 70.4 dB	.186 HZ

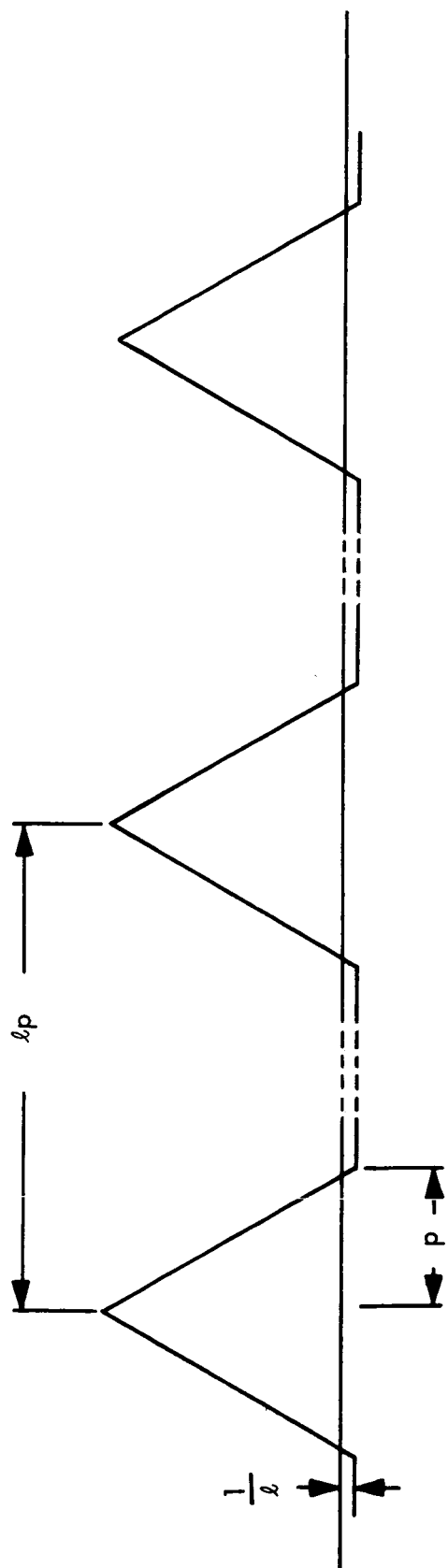


Figure 1. PN Sequence Autocorrelation Function

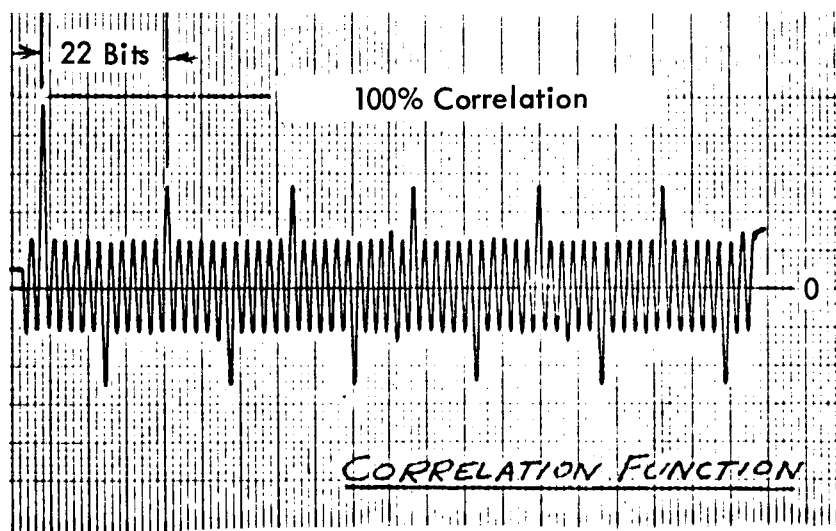


Figure 2. Measured Correlation Function of Apollo Pseudo-Random Range Code (Reference 4)

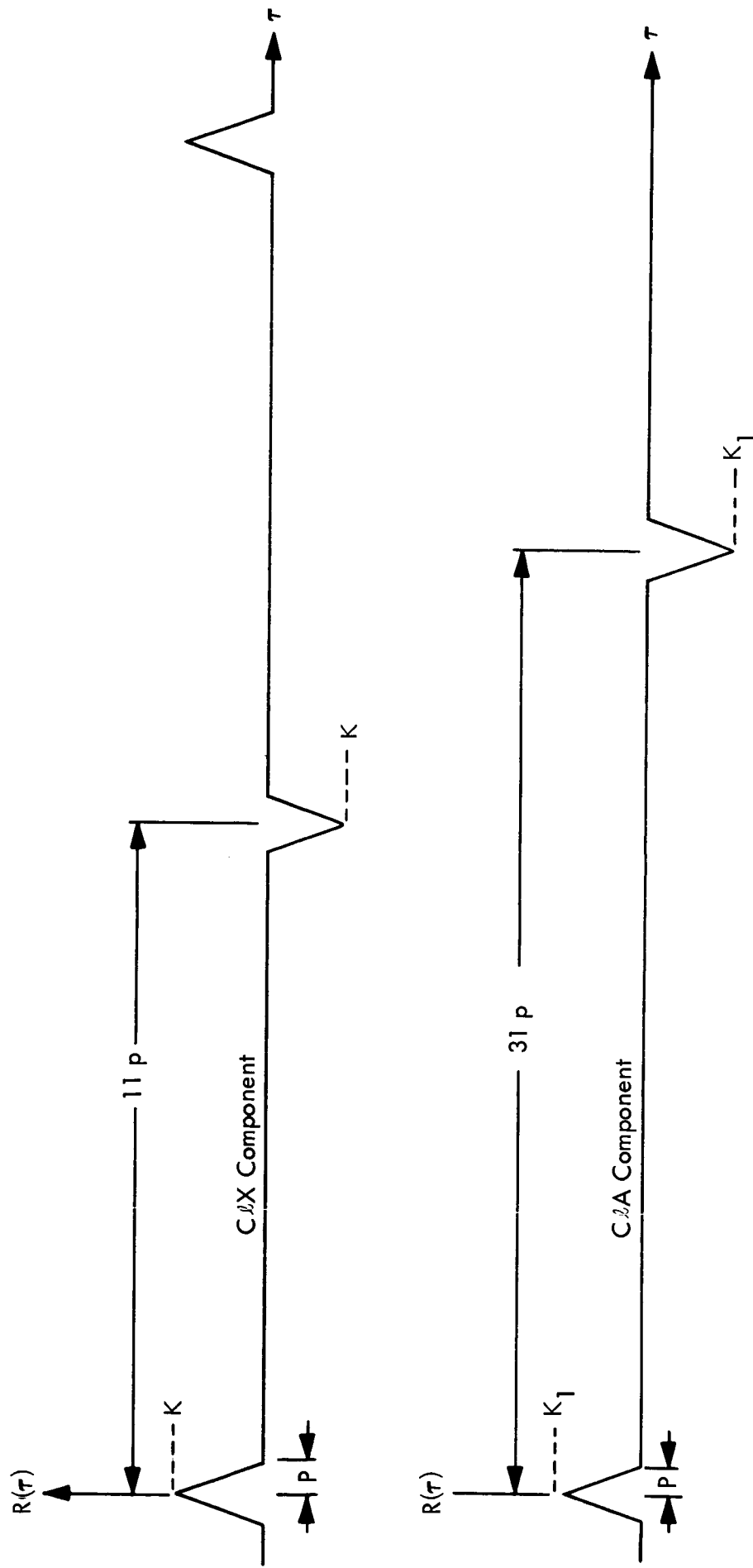


Figure 3. Sketch of Correlation Functions for X and A Components.

$$C_{\mathcal{L}} \oplus \overline{X} [AB+AC+BC]$$

[illegible]

$$\rho = \frac{\text{Number Zeros} - \text{Number Ones}}{\text{Number Zeros} + \text{Number Ones}} = \frac{320 - 192}{320 + 192} = \frac{1}{4}$$

Legend: □ Denotes Possible Occurrences

FIGURE 4. KARNAUGH CHART

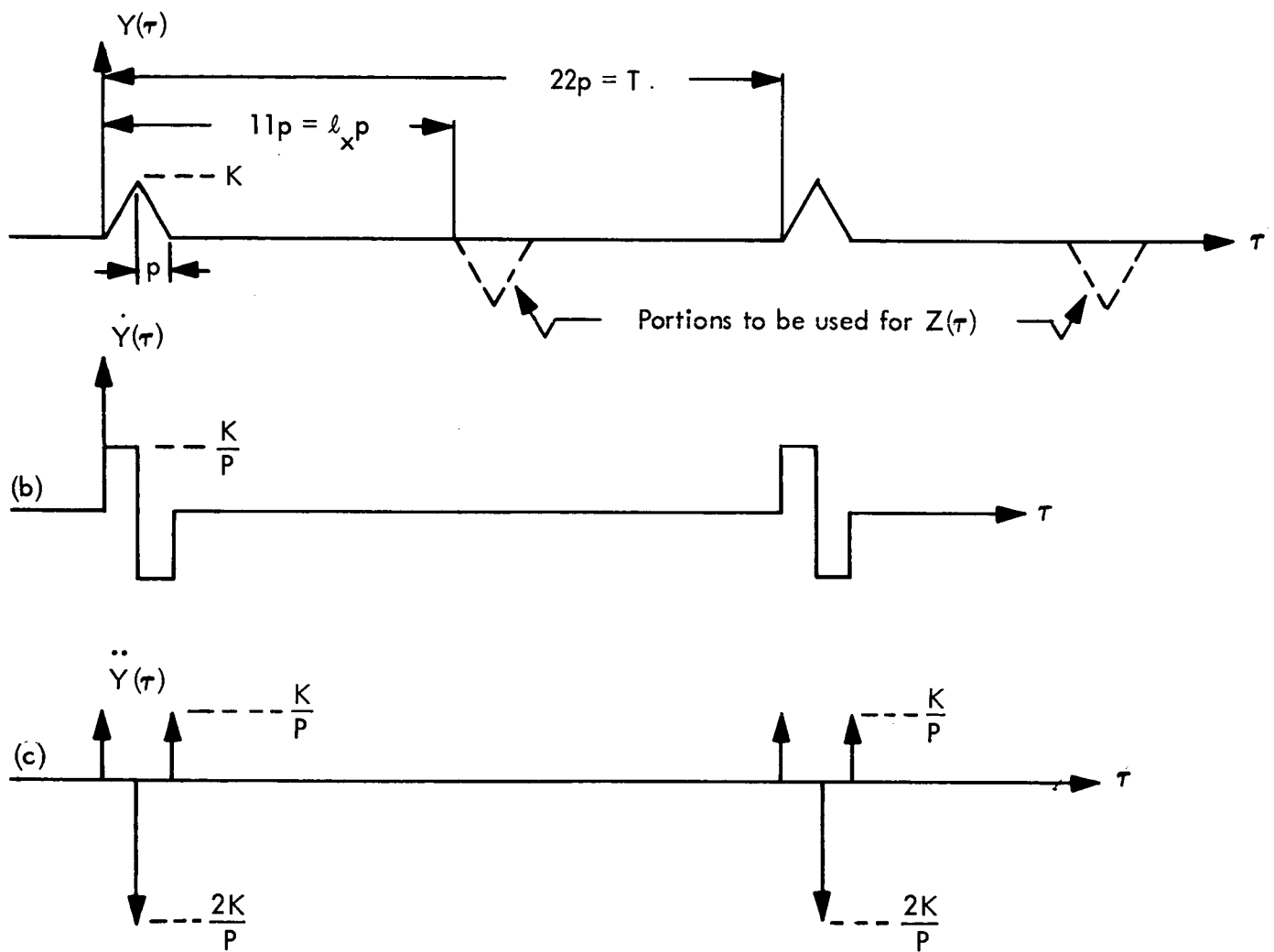


Figure 5. Differentiation of $Y(\tau)$, the Positive Portion of $c\ell X(\tau)$, to Accomplish the Fourier Transform.

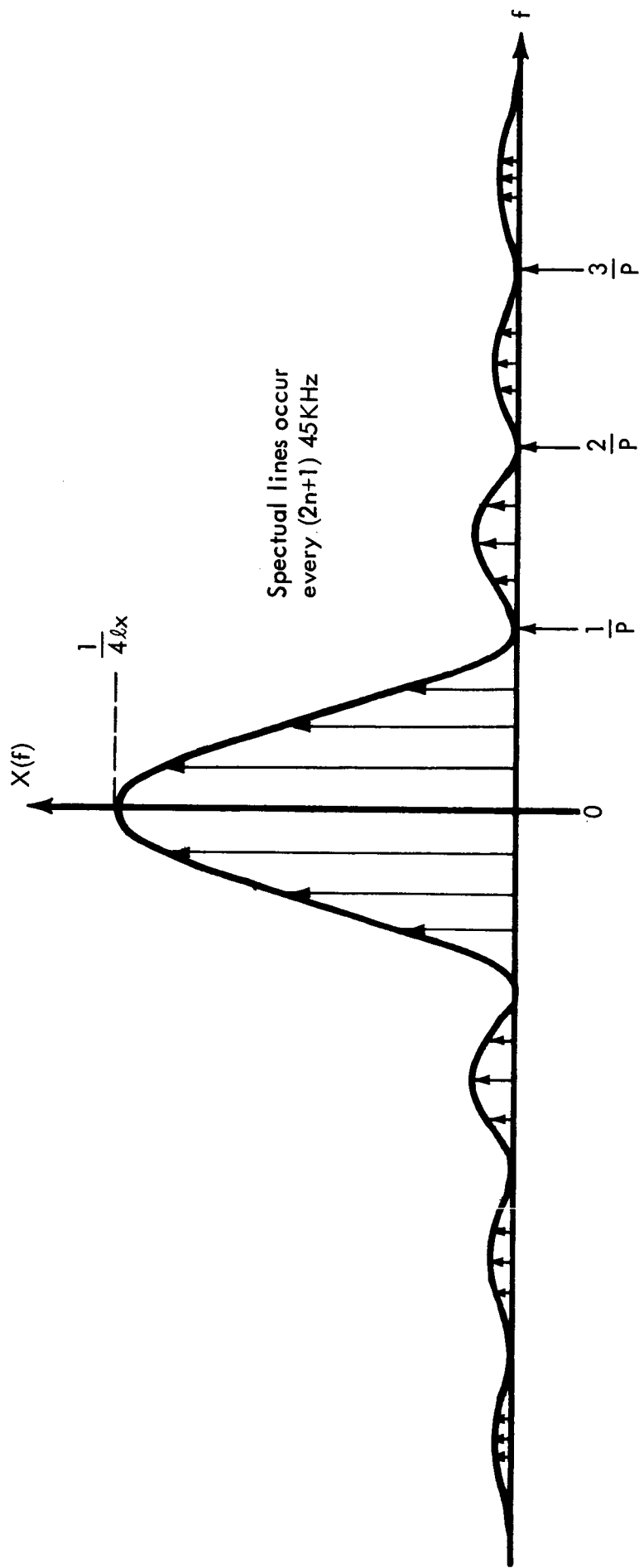


Figure 6. Spectrum Resulting from X Component on Table 1 (Sketch-not to scale)

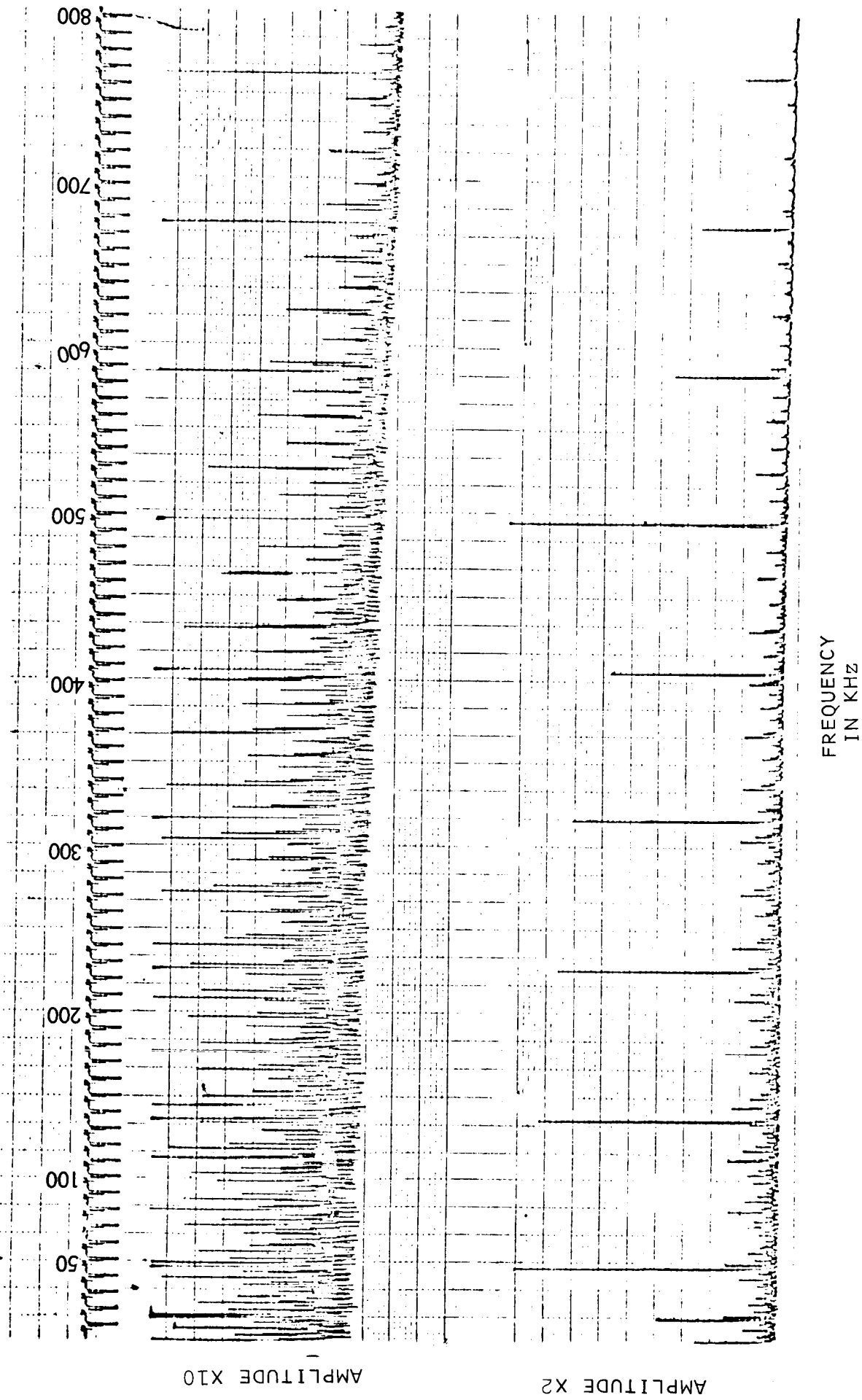


Figure 7. Measured Apollo Psuedo-Random Range Code Base Band Spectrum (Reference 4)

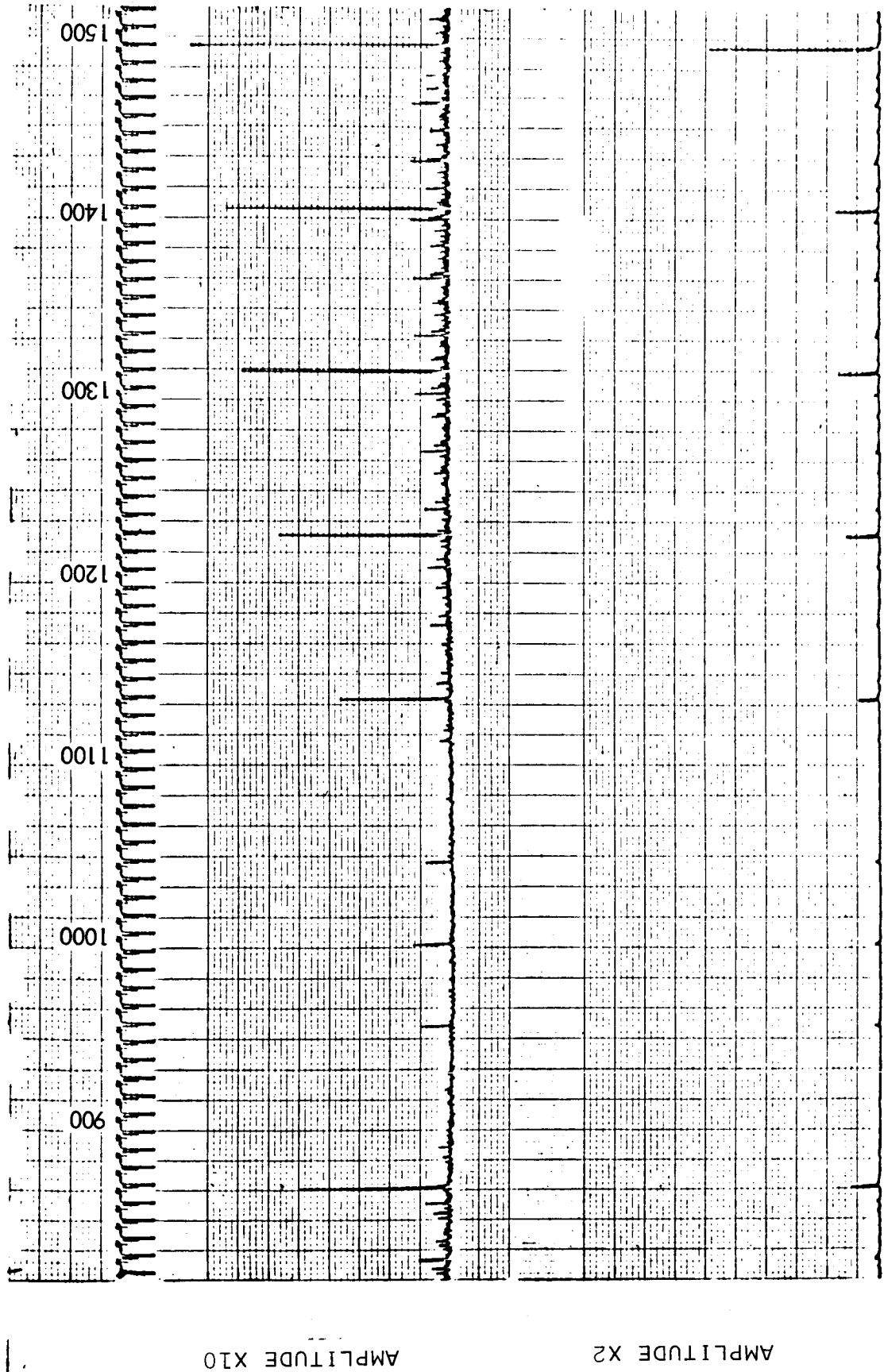


FIGURE 8. MEASURED APOLLO PSUEDO-RANDOM RANGE CODE BASE BAND SPECTRUM (REFERENCE 4)

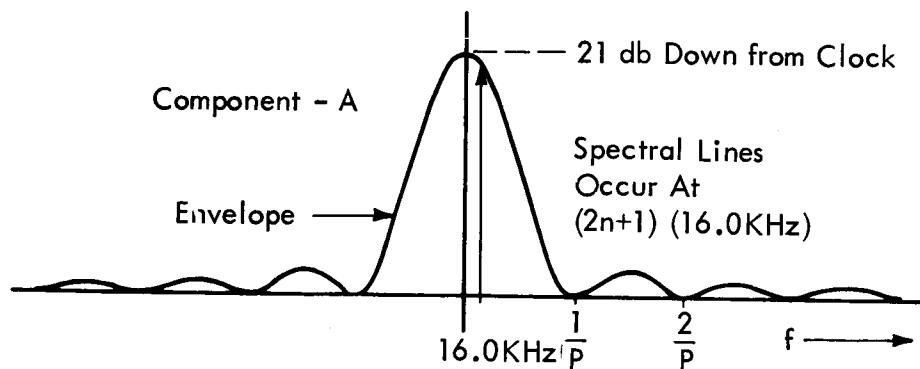
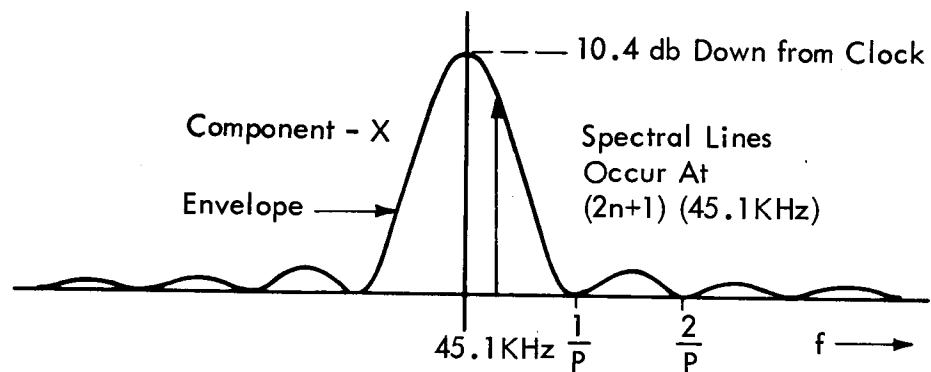
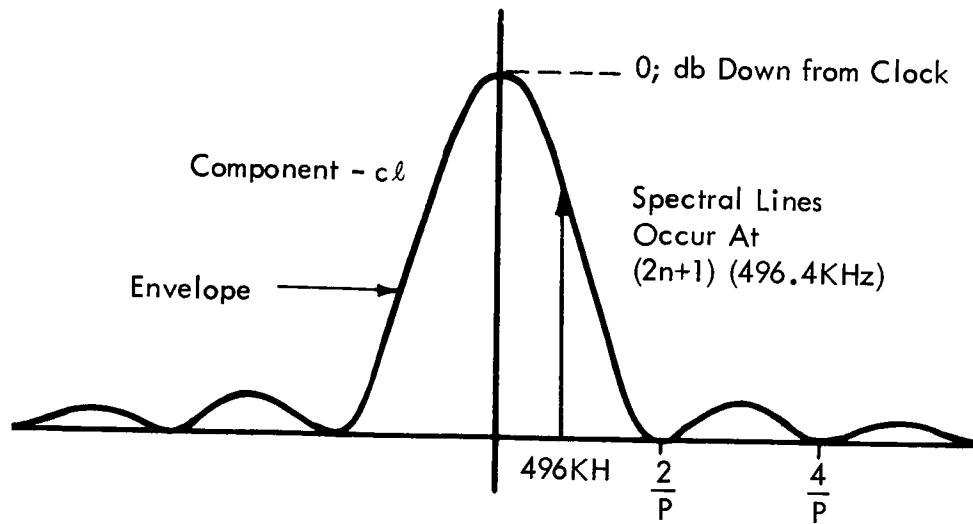


Figure 9. Spectral makeup of Apollo Range Code Components: cl , X and A (only one line shown for clarity purposes — not to scale)

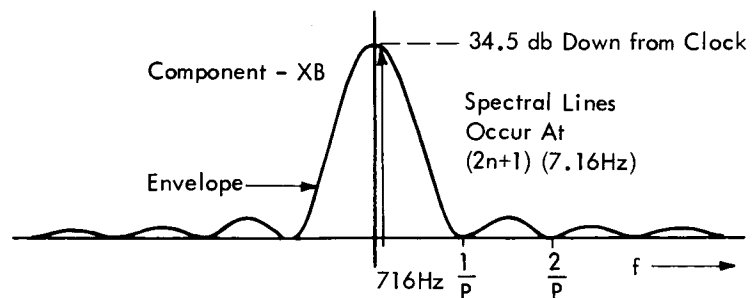
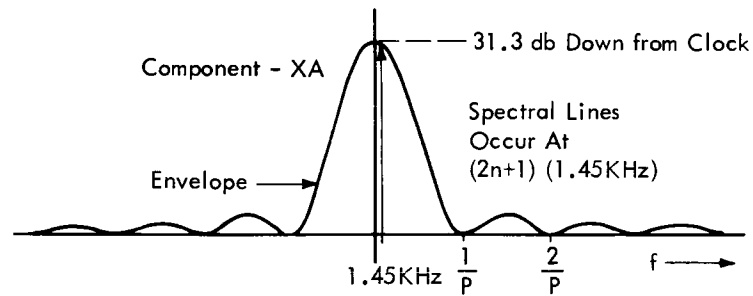
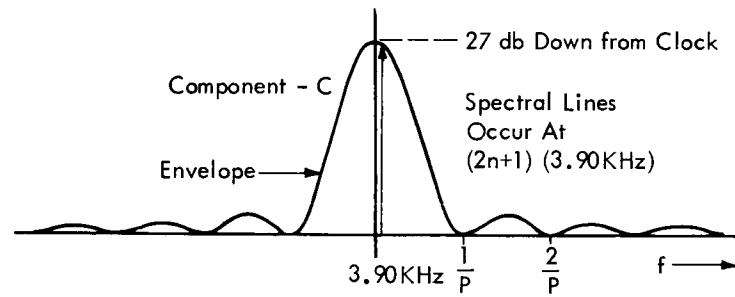
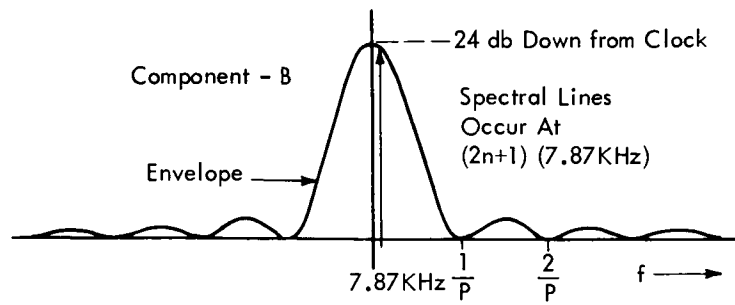


Figure 10. Spectral makeup of Apollo Range Code Components: B, C, XA, and XB (only one line shown for clarity purposes — not to scale)

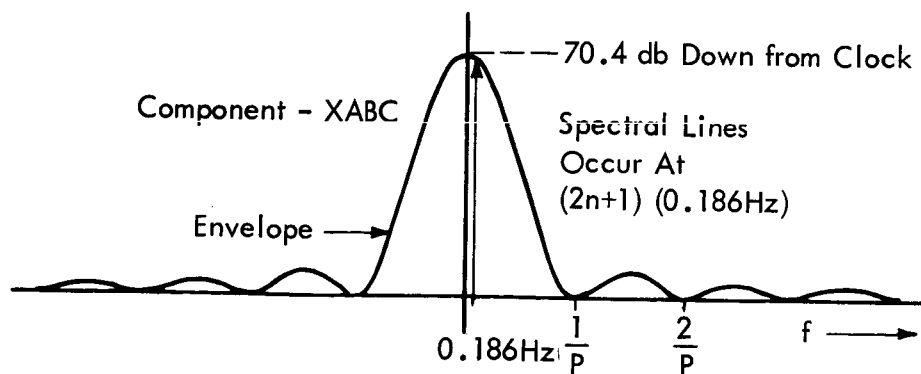
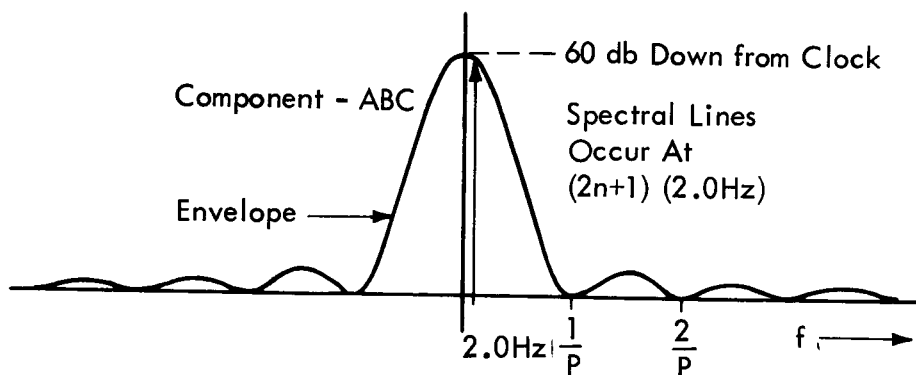
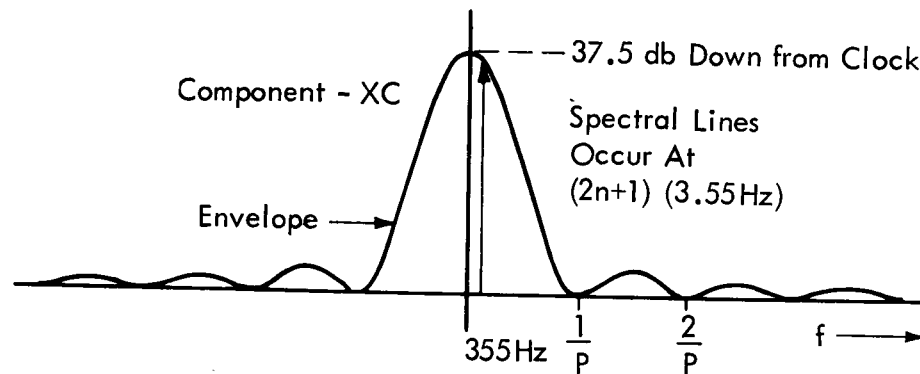


Figure 11. Spectral makeup of Apollo Range Code Components: XC, ABC and XABC (only one line shown for clarity purposes — not to scale)

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